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ABSTRACT

A taxonomy for the classification of fourth-grade mathematics content including tests, workbooks, textbooks, handouts, worksheets, and classroom instruction is presented. Three factors are used to define the taxonomy: mode of presentation, with three levels; nature of the materials, with thirteen levels; and operation, with twelve levels. The definitions of the levels of each factor are given as well as conventions used in conjunction with each factor and examples for each level. Sample test questions are classified and the classification explained. A set of practice items and a key are also presented. (MP)

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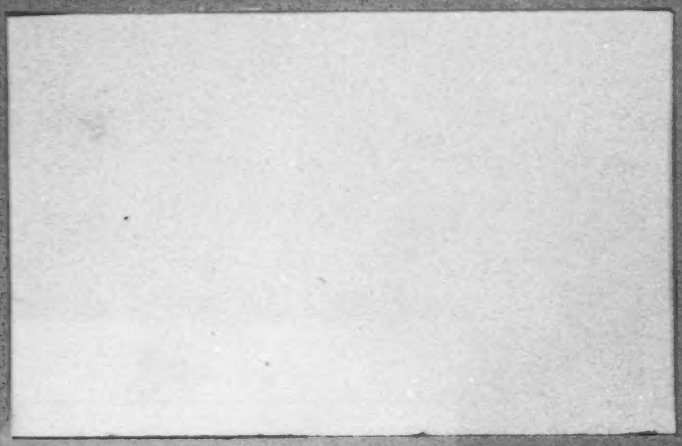

Institute for Research on Teaching

College of Education — Michigan State University

SE 025 332

Research Series No. 4

TRAINING MANUAL

FOR THE CLASSIFICATION OF THE CONTENT

OF FOURTH GRADE MATHEMATICS

by William H. Schmidt,
Andrew C. Porter, Robert E. Floden,
and Donald J. Freeman

Technical Manual

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TRAINING MANUAL
FOR THE CLASSIFICATION OF THE CONTENT
OF FOURTH GRADE MATHEMATICS *

Purpose

The purpose of this manual is to present a taxonomy for the classification of fourth-grade mathematics content and to provide training in the use of that taxonomy. The taxonomy was designed to classify the content of various curricular materials in the field of mathematics including tests, workbooks, textbooks, handouts, and worksheets as well as to classify the content of actual classroom instruction. In some respects the taxonomy may appear to be unrealistically detailed, while in other respects it may appear to gloss over important distinctions. An attempt was made to strike a balance between these two extremes, employing as the guiding principle that classification should be made at a level sufficient to inform curricular decisions.

Development of the taxonomy began with four independent, unstructured content analyses of the items on the Stanford Achievement Test (SAT), grade four. These analyses were pooled to form a first version of the taxonomy. Through an iterative process of applying a current version of the taxonomy to the SAT and then revising the taxonomy when the need was apparent, the present version evolved. Thus far, the taxonomy has been used to analyze the content of four major standardized tests at the fourth-grade level.

Taxonomy

Three factors were used to define the taxonomy: (1) mode of presentation, (2) nature of the materials, and (3) operation. There were three different modes of presentation, 13 different levels of material, and 12 different operations. Conceptually, the taxonomy is formed by the in-

*This manual was developed by William H. Schmidt, Andrew C. Porter, Robert E. Floden, and Donald J. Freeman, senior researchers with IRT's Outcome Measurement group.

tersection of the three factors, which results in 468 possible cells for classification (see Figure 1).

The remainder of this manual elaborates on the factors by specifying: (1) the definitions of the levels of each factor, (2) conventions used in conjunction with each factor to eliminate certain ambiguities that might arise, and (3) examples for each level. The taxonomy and this manual are currently designed for the classification of test items. The verbal descriptions in this text should be interpreted and used with this in mind.

Factor I -- Mode of Presentation

Mode of presentation concerns the format in which content is presented. There are three levels to this factor: (1) graphs, figures, tables, or physical objects, (2) operations specified, and (3) operations not specified (story problems).

Level 1: Graphs, Figures, Tables, or Physical Objects

If any of the information needed to solve the problem is presented in pictorial or physical form, e.g., tables, graphs, figures, or physical objects, then the item is classified at this level. The key point is that information presented in this way must be essential to the solution of the problem and must not be available in any other form.

The following are two examples of items which would be classified at Level 1:

A.

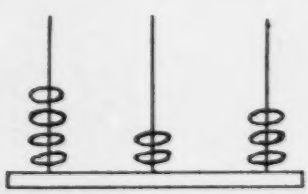


Figure 1

Content Classification of _____

By _____ Date _____

MODE OF PRESENTATION

Nature of the material / Type of Operation		Graphs, Figures, Tables, or Physical Objects												Operations Specified												Operations not Specified (Story)											
		1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	11	12
Whole Numbers	single digit																																				
	single digit and multiple digit																																				
	multiple digit																																				
Fractions	single																																				
	multiple																																				
Decimals																																					
Percentages																																					
Alternative Number Systems																																					
Place Value																																					
Sentences	Number																																				
	Algebraic																																				
Essential Units of Measurement																																					
Geometric Figures																																					
Other																																					

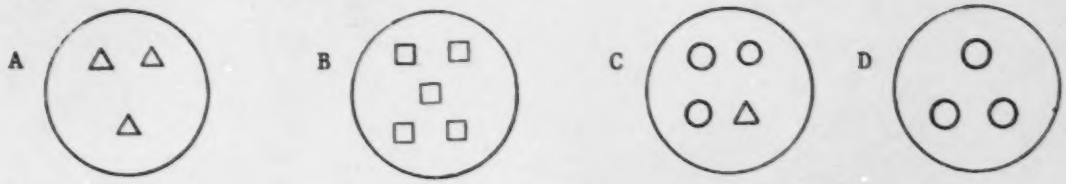
Operations

1. Add
2. Subtract without borrowing
3. Subtract with borrowing
4. Add or subtract fractions without common denominator
5. Multiply
6. Divide without remainder
7. Divide with remainder
8. Combinations
9. Grouping
10. Identify equivalents
11. Identify rule (order)
12. Identify terms

This value is

a) 234
b) 423
c) 432
d) 324

B. Which of the sets below are equivalent?



1) C and D
2) A and D
3) A and C
4) A and B

For both items the information presented pictorially is necessary to answer the question.

Level 2: Operations Specified

For an item to be classified at this level of Factor I, two conditions must be met: (1) the essential information of the item is not presented in graphic form and (2) the mathematical operations involved in the solution are explicitly stated as a part of the problem itself, i.e., the student does not need to infer the operations involved.

Two items which would be classified in this way are:

A. 80,000 + 5,000 + 40 =

a) 85,040
b) 85,400
c) 80,540
d) 854

B. Which of the numbers represented below is nearest in value to 2500?

a) 250
b) 2,498
c) 2,510
d) 2,599

In both cases, the operations that need to be performed are explicitly stated. For item B the question specifies that the student is to compare the values in a) through d) with 2,500, by asking "which is nearest in value?" (See Convention 3, p. 5).

Level 3: Operations Not Specified

Items classified at this level must satisfy two conditions: (1) the essential information of the item is not presented in graphic form, and (2) the mathematical operations involved in the solution of the item are not explicitly stated. For a student to solve the required problem, s/he must infer from the item the necessary operation or set of operations that must be performed. An item typically classified in this way is the traditional story problem. Two examples are as follows:

A. If each box has 8 crayons, how many crayons are in 4 boxes?

a) 12
b) 32
c) 48
d) 20

B. Which of these sets of coins has the greatest value?

a) 1 quarter
b) 23 pennies
c) 4 nickels
d) 3 dimes

To answer question A the student must infer that the multiplication of 4 and 8 is the correct mathematical operation. To answer question B the student must infer that the number of each kind of coin listed must be multiplied by its appropriate value in cents before any comparison is made.

Conventions for Using the Taxonomy for Factor I

1. An item should be classified under Level 2 (operations specified) if, and only if, all the operations necessary to solve the problem are explicitly stated. If any of the operations need to be inferred, even if others are explicitly stated, then the item is classified at Level 3 (operations not specified).
2. If an item presents an algebraic sentence which must be solved for the unknown, the operation (although opposite the one given) is assumed to have been specified and the item is classified at Level 2.
3. Items calling for response to an explicit comparison are classified at

Level 2 if all other comparisons required are explicit. Examples of explicit comparisons are:

(a) which is greater (larger, smaller, more, less, longer, shorter, etc.)

(b) which is nearest in value

4. Items which involve determining the missing elements in a number series are to be classified at Level 3, since the operations needed to determine the next number in the sequence must be inferred from the numbers given. The only exception to this convention is that a number sequence which is merely a counting sequence is classified at Level 2. Two examples to clarify this convention are:

A.

3	8	13	18	

What number comes next?

a) 23
b) 19
c) 20
d) 22

B. What are the missing numerals in the diagram below?

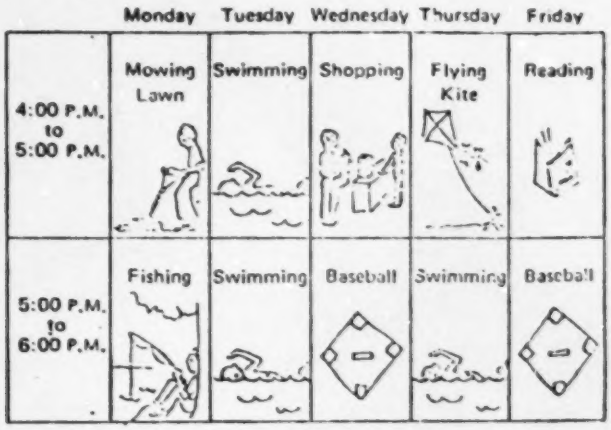
7		9	10	

a) 6, 8
b) 6, 12
c) 8, 11
d) 11, 12

Item A would be classified at Level 3 while item B would be classified at Level 2.

5. If an item focuses on non-quantitative graphical information, it should be classified under "other" which appears at the bottom of the taxonomy (see Figure 1). An example of this is:

Every week Fred makes a plan for spending his time after school. This was his plan for one week:



What did Fred do on Monday between 5 p.m. and 6 p.m.?

1) cut the lawn
2) went fishing
3) went swimming
4) played baseball

6. In classifying items, pay attention only to information essential to the specification of the problem. In most cases this means that the answer to the item as well as the distractors can be ignored in the classification of the item. In some cases, however, some of the essential information is given in the answer or the alternatives or both, in which case such essential information must be considered in the classification. (Convention 6 applies to the taxonomy as a whole, i.e., not only to Factor I but also to Factors II and III.)

Factor II -- Nature of the Material

While Factor I was concerned with how the material was presented to the student, Factor II is concerned with the nature of the material itself, i.e., does the problem deal with whole numbers, fractions, decimals, or units of

measurement? There are 13 levels to Factor II, although Figure 1 seems to suggest 14 since "other" is included as a row in the matrix. As noted previously, the category "other" is not a level of Factor II but rather is used to place items which are not classifiable on one or more of the three factors.

Before looking at the levels of Factor II, a general point must be made about the nature of the material in mathematics problems. For our purposes, there appear to be three different types of material which, separately or in combination, make up fourth grade mathematics problems: (1) quantities, as they are simply and commonly expressed (including whole numbers, fractions, decimals, and percentages), (2) geometric figures, and (3) quantities expressed in ways different from the first category, (e.g., Roman numerals, algebraic and number sentences, measurement units, and place value). By far the most common of these is the first -- quantities -- and so most math problems include quantities, even though in many instances these quantities are embedded in problems which involve greater complexities. For example, a problem might concern determining how many quarts there are in one gallon. The quantities here are all single-digit whole numbers, but the solution necessitates knowledge about units of measurement as well. To classify such an item as dealing with whole numbers misses the most important aspect of the problem. Hence, such items are classified not according to the nature of the quantity involved (i.e., whole numbers, fractions, etc.), but according to the nature of the complexities for which these digits have been employed. For this reason, the definitions of Levels 1-7 which deal with quantities and nothing more suggest not only what the level is, but also what it is not. This increases the complexity of the taxonomy but is necessary for logical completeness, i.e., to make Levels 1-7 of this factor mutually exclusive from Levels 8-13.

Level I: Whole Numbers - Single Digits

For an item to be classified at this level: (1) all quantities used to define the problem must be single-digit whole numbers (i.e., 0 through 9), and (2) several additional conditions must also be satisfied which are exclusive (i.e., they describe additional characteristics which the item must not have in order to be classified at this level. (These conditions for exclusion are the same for Levels 1-7 of this factor. They are presented in detail here and will be referred to in subsequent sections in abbreviated form as the "exclusive conditions.")

The exclusive conditions are: (1) the item does not present the quantities in the form of an explicit algebraic sentence which must be solved for the problem to be answered, (2) the item does not indicate that the answer must be expressed in algebraic or number sentence form, (3) the item is not obviously designed to test whether a student can recognize the different values digits have when placed in different positions within a number, (4) if the item involves units of measurement, no knowledge of the units themselves are necessary, (5) the quantities in the problem are not expressed in alternative bases or number systems, and (6) the item is not designed to identify a geometric figure or its properties.

The exclusive conditions divide the 13 levels of Factor II into two groups: the "quantities" (Levels 1-7) and the "more-than-quantities" (Levels 8-13). Still, when classifying items within the two major groupings, the levels may not appear on the surface to be mutually exclusive. The levels have, therefore, been ordered according to ascending priority, and so the highest level which characterizes the item is the level at which it should be classified. This is the substance of Convention 1 on page 17 and is included here to help in the reading of this section.

Two examples of items classified at Level 1 are:

A. On pet day at school nine children brought 3 dogs, 4 rabbits, 2 turtles, and 2 hamsters. How many pets did we have all together?

a) 7
b) 20
c) 11
d) 9

B. 6 + 9 =

a) 12
b) 15
c) 13
d) NG

For both items, all the essential quantities involved in the specification of the problem are represented by single-digit numerals. Notice that the answers and some of the incorrect alternatives for both A and B are not single-digit numerals. Since this information is not necessary for the solution of the problem, the items are classified as single-digit whole numbers. (For item B, see Convention 2, p. 19.)

Level 2: Whole Numbers - Single and Multiple Digits

Level 2 is defined in the same way as Level 1, except that the numerals in the problem may include both single and multiple digits. However, items having both single and multiple digits must have one and only one multiple-digit numeral specified in the problem to be classified at Level 2. Items classified at Level 2 must also meet the exclusive conditions.

Two examples are:

A. Susan had 14 pine cones. She gave Carol 2 and Patty 4. How many does Susan have now?

a) 10
b) 6
c) 8
d) 20
e) NH

B.
$$\begin{array}{r} 3 \\ 8 \\ +11 \\ \hline \end{array}$$

a) 22
b) 27
c) 12
d) NG

Level 3: Whole Numbers - Multiple Digits

This level is similar to Levels 1 and 2 in that: (1) the exclusive conditions must again be met, but differs in that (2) two or more multiple digit quantities are involved in the specification of the problem. (Note: Those items in which both single- and multiple-digit numerals are specified and where two or more of these quantities are multiple digits are classified at this level.)

Two examples of items classified at level 3 are:

A. A national park is 450 miles from our home. If we go 240 miles the first day, how many miles from the park will we be?

a) 110
b) 210
c) 690
d) 290

B.
$$\begin{array}{r} 3 \\ 18 \\ +11 \\ \hline \end{array}$$

a) 32
b) 27
c) 42
d) NG

All the quantities in example A are expressed as multiple digits. Although the problem concerns distances expressed as miles, the solution does not necessitate converting miles to any other unit of measurement, so the item is classified at Level 3. Item B is classified at Level 3 since two multiple digits are specified in the problem.

Level 4: Fractions - Single

For an item to be classified at this level, the exclusive conditions must again be satisfied, and one and only one of the quantities specified in the problem must be a fraction (mixed numbers are considered as fractions).

(Note: Consistent with Convention 6, p. 7 and Convention 2, p. 19, for computational problems, the answer to the item is not counted toward the requirement that there be one and only one fractional quantity specified,

e.g., ½ x 1 = ½ involves only one fractional quantity for purposes of classification.)

Two examples of items classified at Level 4 are:

A. What part of this figure is shaded?

a) 1/2
b) 1/3
c) 1/4
d) 1/5

B. If ½ of 6 is 3, than ¼ of 8 is

a) 12
b) 1
c) 4
d) 2

In both A and B, one of the quantities specified is a fraction. Notice that in question A the correct answer must be used to properly classify the item since it provides essential information. For example B, the second fraction "¼" in the item is not essential information for solving the problem.

Level 5: Fractions - Multiple

The definition for this level includes the exclusive conditions given for Level 4 but changes the other requirement to read two or more of the quantities involved in the specification of the problem are fractions.

Two examples are:

A. 12 1/3 + 4 1/4 =

a) 16 1/8
b) 16 7/12
c) 17 5/12
d) 17 3/4

B. 1/5 ÷ 1/5 =

a) 1/25
b) 1/10
c) 2/5
d) 1

Notice that for example A mixed numbers are included in the items which are classified as fractions.

Level 6: Decimals

If one or more of the quantities in an item are expressed in decimal form, e.g., .07 or .932, and the exclusive conditions expressed for all of the previous levels of Factor II hold, the item would be classified at Level 6.

Two such items are:

A. 48.3
-4.8

a) 4.35
b) 4.46
c) 43.5
d) 44.6

B. Sue earned $4.50, $3.25, and $2.25. If you add these quantities together, how much did she earn altogether?

a) $10.75
b) $ 9.00
c) $10.00
d) $11.00

For both items, the basic quantities are expressed as decimals. (For clarification on the use of money in item B, see Convention 5a, p. 20.)

Level 7: Percentages

An item is classified at this level if: (1) one or more of the quantities in the item are given as percentages, and (2) the exclusive conditions hold.

Two examples are:

A. What percent of this region is shaded?

a) 50
b) 10
c) 25
d) 100

B. Which has the smallest value?

a) 85%
b) 75%
c) 60%
d) 55%

Level 8: Alternative Number Systems

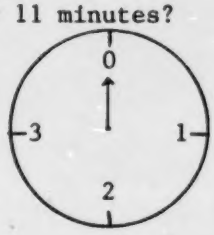
If an item deals with a number system other than Arabic, the problem is classified at Level 8 of Factor II no matter what type of quantities are being dealt with. Items dealing with Roman numerals are common examples.

Two examples are:

A. VI =

a) 11 c) 51
b) 4 d) 6

B. Below is a picture of a clock with one hand. The hand takes 4 minutes to go around once. If the hand starts at 0, where will it be after 11 minutes?



a) 0 c) 2
b) 1 d) 3

Problem A concerns Roman numerals, while B essentially involves representing the numeral 11 on the four-digit clock face which can be thought of as an alternative number system.

Level 9: Place Value

An item is classified at this level if the primary purpose of the item is to determine whether the student can recognize the different values digits have when placed in different positions within a numeral. An example is to recognize that the digit "1" in "10" signifies ten, but that the digit "1" in "100" signifies one hundred. If the numbers included in the problem each contain a single non-zero digit and those non-zero digits vary dramatically as to their assumed numerical value, the item is classified as place value (see example B below).

Three examples of items dealing with place value are:

A. In which numeral is 5 in both the hundred place and the ones place?

a) 4,755
b) 4,575
c) 4,557
d) 5,475

B. 80,000 + 5,000 + 40 =

a) 85,040
b) 85,400
c) 80,540
d) 854

C.

This value is

a) 234
b) 423
c) 432
d) 324

All three items are concerned with recognizing the value a numeral assumes when placed in different positions. Item B, although dealing with the addition of multiple digits, is classified here since the intent of the item deals with recognizing the position each digit takes in the final number.

Level 10: Sentences - Number

If the answer to a problem is expressed as a number sentence (and not as a solution of that number sentence), then the item is classified at Level 10 of Factor II. This is true no matter what type of numerals are used in the item or number sentences. A number sentence is one in which either: (a) there are no unknowns, or (b) the unknown, if present, is isolated on one side of the equal sign. For items classified at this level, the answer must be expressed in sentence form. The formulation of the problem stem is irrelevant. Expressions such as 2 + 3, 6 - 2, or 501 x 4, which are not technically number sentences, are still classified as such for purposes of this taxonomy.

Two examples of items classified in this way are:

A. $7 + (5 + 4) =$

a) $(7 + 5) + (7 + 4)$
b) $(7 + 5) + 9$
c) $(3 + 4) + (3 + 2)$
d) $(7 + 5) + 4$

B. There were 13 children waiting to cross the road. Eight of them ran across. How many were still waiting to cross?

a) $\square \times 8 = 13$
b) $13 - 8 = \square$
c) $13 + \square = 8$
d) $13 + 8 = \square$

Both items are classified at this level because the answer itself is in terms of a number sentence, e.g., in A you do not need to know the answer is 16, but that it can be expressed as $(7 + 5) + 4$, and in B you do not need to know 5 is the answer, but that the solution is expressed in sentence form as $13 - 8 = \square$. In B, alternatives a) and c) are algebraic sentences instead of number sentences, but since they are distractors they are ignored in classifying the item.

Level 11: Sentences - Algebraic

An item is classified at this level of Factor II if: (1) the answer to the item is expressed in the form of an algebraic sentence, or (2) the problem itself involves an algebraic sentence and the solution to the problem is found by determining the value of the unknown in that sentence. An algebraic sentence is defined as a sentence involving one or more unknowns with at least one unknown not isolated on one side of the equal sign.

Two examples are as follows:

A. What will 30 books cost for one class? You can answer this if you know

a) the size of the class
b) the names of the books
c) which room will use them
d) the cost of each book
e) who sells the book

B. If $19 \div a = b$, then _____

a) $b + a = 19$
b) $a \div b = 19$
c) $b \times a = 19$
d) $b \times 19 = a$

For both items the answer is not a numerical solution but an equation by which the solution is derived. In example A, the answer is not an algebraic sentence, but rather the variables needed to construct an equation. Nevertheless, the item is classified at Level 11 since the reasoning process is similar to that of identifying the right equation.

Level 12: Essential Units of Measurement

If a problem deals with quantities expressed in different units of measurement and the quantities must be transformed from one unit of measurement to another in order to solve the problem, then the item is classified under Level 12 of Factor II, regardless of the type of quantities involved (e.g., whole numbers, fractions, or decimals). The key to correctly classifying items as units of measurement is to determine if transforming quantities from one metric into another is essential.

Two examples are:

A. How many weeks away from May 5 is May 26?

a) 2
b) 4
c) 3
d) NG

B. If candy costs 10¢ an ounce, what will a pound cost at that price?

a) $1.00
b) $1.20
c) $1.25
d) $1.60
e) NH

For item A, in order to solve the problem, it is necessary to recognize that there are 7 days in a week, and for item B, one needs to know that ther are 16 ounces to a pound. Since the transformations involving different units of measurement are necessary, these items are classified at Level 12 of Factor II instead of at single and multiple digits or decimals, respectively.

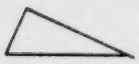
Level 13: Geometric Figures

If an essential part of a problem is to identify a geometric figure or its properties, the item is classified under Level 13 of Factor II. This

level should not, however, be confused with Level 1 of Factor I, which specifies that the mode of presenting information is graphical. For example, a problem can have some data presented in the form of a geometric figure, but the mathematical material needed for the solution of the problem is numerical and does not deal with the actual figure or its properties. Such an item should not be classified at this level. (See item A under Level 4 on page 12 as an example in which the mode is geometric but the material itself is numeric.) This is different from a case where the mode is geometric and the solution of the problem involves the figures themselves or their properties. Such an item would be classified at Level 1 on Factor I as well as at Level 13 on Factor II. Most geometry problems would fall into this latter category.

Two examples are:

A. Which triangle is similar to triangle KLM?

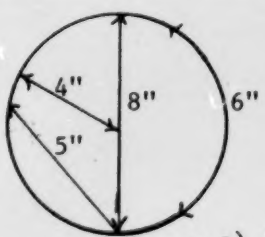
a) 

c)

b)

d)

B. How long is the diameter of the circle shown below?



a) 4"
b) 5"
c) 6"
d) 8"

Both items involve geometric figures and their properties. Item A is classified at Level 13 since it requires knowledge of similar triangles, while item B is classified at Level 13 since the property of the diameter of a circle is critical to answering the item.

Conventions for Using the Taxonomy for Factor II

1. If an item involves material, some of which is classifiable at one level of Factor II and some of which is classifiable at a different level, the item should be classified at the highest level (where the order is as specified in the taxonomy in Figure 1, i.e., the further down the list, the higher the priority). For example, if an item deals with both decimals and single digit whole numbers, the item would be classified at the level decimals.

2. For straightforward computational problems such as:

	a) 12	3	a) 14		a) 81
3	b) 14	6	b) 16	$4.\overline{)32.4}$	b) .81
x4	c) 21	+8	c) 17		c) 8
	d) 16		d) 27		d) 8.1

the answer is never used in classifying the item. (This convention is consistent with Convention 6, p. 7, but makes the rule more explicit for a common subset of mathematical problems.)

3. For problems where an intermediate calculation is clearly specified by parentheses or in some other explicit way and results in a multiple-digit number, that multiple number must be considered with all other digits in classifying the item. Several examples are:

   A. $(3 \times 4) + 12 =$ multiple digits

   B. $(3 + 9) + 2 =$ single and multiple digits

   C. $(16 \div 8) + 4 =$ single and multiple digits

   D. $(96 \div 8) + 3 =$ multiple digits

4. For graphs involving non-numeric symbols or characters, treat the characters according to their numerical equivalent.

5. For problems involving money, assume that the students can convert ¢ to $, e.g., they know 59¢ = $.59. Problems involving such conversions are not classified as units of measurement. Additionally:
   a) if all information is given in decimal form, then classify the item as decimals.
   b) if all information is given in cents, then classify the item under one of the whole number levels.
   c) if information is given both in cents and decimal form, then classify the item as decimals. (In this context, the form of the answer is considered as information relevant to the classification of the item.)
6. Problems which require comparing multiple digit numerals, each made up of the same digits, are classified as place value items, e.g., which is larger, 234 or 432?
7. If graphs have only whole numbers explicitly given, and the answers are either whole numbers or expressions like "almost 5," the nature of the material is whole numbers.
8. If the exclusive conditions are met and the essential quantities in a problem are expressed with percent signs or decimals, do not attempt to reason whether the percent signs or decimals make any "real" difference in the problem, just classify them accordingly. For example, $3.00\overline{)24.00}$ is classified as decimals even though the decimals do not make any "real" difference, i.e., $3.00\overline{)24.00} = 8.00$ is the same as $3\overline{)24} = 8$.
9. In classifying the problems involving number series, use only the digits given explicitly in the problem, i.e., do not use the intermediate results arrived at by analyzing the series and calculating the missing digit in the sequence. Also, consider all digits explicitly given in the series as essential information for classification.

10. For items classified at Level 3 of Factor I, i.e., operation not specified, careful attention must be given to classifying them on factor II. The item should be read and the number or algebraic sentence appropriate to the solution of the problem written down as the information is given in the item. A literal transcription of the verbiage contained in the problem into the corresponding equation is needed to properly classify the item. Whether done explicitly or implicitly, this transcribed sentence is the intermediate step necessary to solving the problem. If the intermediate equation (not the solution itself) is a number sentence, then the item is not classified as a number sentence but according to the nature of the material used in the item, i.e., multiple digits, fractions, measurement units, etc. If, on the other hand, the intermediate equation is an algebraic sentence, then this item would be classified on Factor II as an algebraic sentence. If the correct answer to the item is not a numerical solution but the number or algebraic sentence itself, then the item would be classified as the appropriate type of sentence. Since this convention is central to classifying a large percentage of items found on tests, several annotated examples are now given. The examples are:

A. If Joe has 3 quarters and Jack has 3 dimes, how much money do they have?

a) \$.60
b) \$.75
c) \$1.05
d) \$1.50

The intermediate equation is 3 x .25 + 3 x .10 = ?, which is a number sentence. The item is classified as to the nature of the material, and since the conversion of quarters and dimes to cents is necessary, the problem is classified as <u>essential units of measurement</u>.

B. Pencils cost 7¢ each. The best way to find out how much 14 pencils cost is to

a) add 7 and 14
b) subtract 7 from 14
c) multiply 14 by 7
d) divide 14 by 7

The intermediate equation is 7 x 14 = ?, which is a number sentence. Since the answer is expressed not as a numerical solution but the sentence itself, this item would be classified as number sentence.

C. There were 165 different bird pictures in a full set. With the first week's newspaper coupons, the Bird Club got 138 different pictures. How many more pictures were needed for a full set?

a) 27
b) 33
c) 37
d) NG

The intermediate equation would be 165 = 138 + X, which is an algebraic sentence and would be classified as algebraic sentence.

D. The next Saturday, Clay parked 18 cars in his yard, and John parked 15 in his. How many more cars did Clay park than John that day?

a) 5
b) 3
c) 13
d) 8

The intermediate equation is 18 - 15 = x, which is a number sentence. The numerals involved are all multiple digits and so the item would be classified as multiple digits.

E. One Saturday both yards were full. There were 36 cars in all; 19 of them were in Clay's yard. How many were in John's yard?

a) 13
b) 17
c) 23
d) NG

The intermediate equation is 36 = 19 + x, which is an algebraic sentence. The item is therefore classified as an algebraic sentence.

F. Lori and Eve brought in 24 coupons altogether. Each girl brought in the same number. How many did each girl bring?

a) 12
b) 26
c) 48
d) NG

The intermediate equation is $2x = 24$, which is an algebraic sentence. The item is therefore classified as an algebraic sentence.

Factor III: Type of Operation

Factor I dealt with the mode by which the essential information of the item was presented, while Factor II defined the nature of the material itself. Factor III concerns the type of operation which must be applied to the material, no matter how it is presented, in order to answer the question. There are 12 levels to this factor, the first seven of which deal with ordinary arithmetic operations that are easily defined by examples. For the remaining 5 levels, the format of presentation is as given in the previous sections. When classifying items on Factor III, the levels may not appear on the surface to be mutually exclusive since an item may include more than one type of operation. The levels have therefore been ordered according to ascending priority, and so the highest level which characterized the item is the level at which it should be classified. This is the substance of Convention 1 on page 32 and is included here to help in the reading of this section.

Level 1: Add

Examples:

A. $\frac{1}{2} + \frac{1}{2} =$

a) $\frac{1}{4}$ c) 2
b) 1 d) 6

B. $\square - 6 = 14$, what number belongs in the $\square$?

a) 8 c) 20
b) 4 d) 22

Although the stated operation for item B is subtraction, it is classified under addition since it is the operation necessary for solution of the

equation (see Convention 2, p. 32). (For item A, the fractions being added have a common denominator and are classified here instead of at Level 4.)

Level 2: Subtract Without Borrowing

Examples:

A. The next Saturday, Clay parked 18 cars in his yard, and John parked 15 in his. How many more cars did Clay park than John that day?

a) 3
b) 4
c) 13
d) NG

B. 247
-133

a) 370
b) 104
c) 114
d) 124

Level 3: Subtract With Borrowing

Examples:

A. 1204
-978

a) 225
b) 326
c) 336
d) 226

B. One Saturday both yards were full. There were 36 cars in all, and 19 of them were in Clay's yard. How many were in John's yard?

a) 13
b) 17
c) 23
d) NG

For item B, see Convention 2, p. 32.

Level 4: Add or Subtract Fractions Without Common Denominator

Examples:

A. ½ + ¼ =

a) 1/8
b) 1/6
c) 3/4
d) 8

B. 12 1/3
+4 1/4

a) 16 1/8
b) 16 7/12
c) 17 5/12
d) 17 3/4

Level 5: Multiply

Examples:

A. 3,057
x 6

a) 18,013
b) 18, 342
c) 18,432
d) 183,042

B. A crate holds 12 quart boxes of berries. One afternoon the boys picked enough boxes to fill 3 crates. How many boxes in all did the boys pick that afternoon?

a) 15
b) 24
c) 35
d) NG

Level 6: Divide Without Remainder

Examples:

A. $5\overline{)330}$

a) 9
b) 15
c) 66
d) 71

B. If stamps are 10¢ each, how many can you buy for 40¢?

a) 50
b) 30
c) 10
d) 4

For item B, see Convention 2, p. 32.

Level 7: Divide With Remainder

A. $9\overline{)183}$

a) 2 R3
b) 9 R3
c) 12 R3
d) 20 R3

B. $4\overline{)83}$

a) 20
b) 20 3/4
c) 22
d) 21

Level 8: Combinations (2 or more different types of operations)

This level of Factor III is for items in which: (1) more than one of the arithmetic operations, i.e., add, subtract, multiply, and divide, is specified in the same problem, and (2) there is no explicit grouping of

the operations, i.e., no parentheses.

Two examples are as follows:

A. If n x 8 = 10 x 4, then n is equal to

a) 1 c) 6
b) 5 d) 7

B. 4 x 12 ÷ 3 = □

a) 8 c) 4
b) 16 d) NG

For item A, both sides of the equation involve multiplication, but once the multiplication on the right hand side of the equation is completed, the problem essentially becomes algebraic (8n = 40) and such an item is classified not according to the operation given, i.e., multiplication, but according to the operation opposite the one given -- division in this case. Thus, both operations are present in the solution of this problem.

Level 9: Grouping

For an item to be classified at Level 9: (1) multiple operations (but not necessarily different ones, i.e., they could be the same operation repeated more than once) must be present in the same problem, and (2) an order of calculation must be explicitly stated by the presence of parentheses.

Two examples are:

A. 10 - (6 + 4) =

a) 0 c) 8
b) 4 d) 10

B. (3 + 4) + 7 =

a) 16 c) 14
b) 13 d) 18

Item A is classified at this level rather than at Level 8 because of the grouping implied by the parentheses. Item B involves only one type of operation, but it is repeated more than once and parentheses are present in the

problem. Note, however, that although the parentheses are not important to the calculations, the item is still classified at Level 9.

Level 10: Identify Equivalents

Items are classified at Level 10 of Factor III if the solution to the problem demands recognition of an alternative but equivalent way to represent the material contained in the item. Another way of stating this is that the solution to the problem is an expression or quantity which is mathematically equivalent to an expression or quantity contained in the item but is expressed in a different way (or form). Several examples of equivalent forms are given in the following table.

Form of the material as given in the item	Equivalent form as given in the solution
1. Graphical or pictorial (including pictorial representations of geometric figures or sets)	Numbers or number sentences representing the graphical material
2. Verbal description of a mathematical problem (story problem)	Number sentence representing the problem
3. Numbers expressed in words (e.g., ninety-two)	The corresponding numbers themselves (e.g., 92)
4. Numbers themselves	The same numbers expressed in words
5. Number sentences	Alternative number sentences equivalent in value
6. Numbers expressed in other number systems (e.g., Roman numerals such as VI)	The same number expressed in Arabic form (e.g., 6)
7. Place value problems where the number is represented in additive form (e.g., 80,000 + 5,000 + 40)	The number itself (e.g., 85,040)

Several examples are now presented:

A. $7 + (5 + 4) =$

a) $(7 + 5) + (7 + 4)$
b) $(7 + 5) + 9$
c) $(3 + 4) + (3 + 2)$
d) $(7 + 5) + 4$

B. Paul had 23¢. He spent 8¢ at the candy store. Which number sentence below can you use to find how much money Paul has left?

a) $23 + 8 = \square$
b) $8 + 23 = \square$
c) $23 - 8 = \square$
d) $8 - 23 = \square$

C. Three of these are equal. Which one is not equal to the other three?

a) $(4 \times 3) + (2 \times 3)$
b) 6×3
c) $(4 + 2) \times 3$
d) $(2 \times 3) + (3 \times 3)$

D.

This was one set. Now it is two sets. This is what happened:

a) $2 + 6 = 8$
c) $8 - 2 = 6$
c) $6 + 2 = 8$
d) $6 - 2 = 4$

E.

This value is

a) 234
b) 423
c) 432
d) 324

Items A and C are examples of type 5 in the table. Item B is an example of type 2 and items D and E are examples of type 1. (Note: Although in item C one might perform the arithmetic operations involved in order to determine equivalence, the item is classified at Level 10 since the solution calls for identifying equivalent number sentences.)

Level 11: Identify Rule (Order)

Items which do not explicitly involve any of the arithmetic operations described in the first nine levels of this factor but which require the ordering or seriation of a set of numbers are classified at Level 11 of Factor III. The classical number series problem is an example. Items which ask for the greatest or smallest values from a set of numbers are similarly classified at this level, as are items involving temporal sequence.

The following items are classified as Level 11:

A. What should be next in this series?

57, 64, 71, 78, __?

a) 79 c) 85
b) 81 d) 88

B. Which of these days of the month comes before the 10th?

a) 13th c) 30th
b) 12th d) 3rd

C. Different melons weighed 12 lbs., 10 lbs., 22 lbs., 15 lbs., and 16 lbs. How many pounds did the middle-sized one weigh?

a) 12 c) 15
b) 22 d) 10

D. Tony had 4 more marbles than Larry and Jesse had 3 less marbles than Larry. Which boy had the most marbles?

a) Jesse c) Tony
b) Larry d) can't tell

Item A is a seriation problem. Item B deals with temporal order. Items C and D deal with ordering and then picking, in the case of C, the middle value, and, in the case of D, the largest value.

Level 12: Identify Terms

Two kinds of items are classified at Level 12 of Factor III: (1) items which ask for the definition of a mathematical term or for an illustration

of the term and (2) items which involve one or more of the 11 previously defined operations but depend for solution upon knowledge of uncommon mathematical terms. The reference to "uncommon" in the above statement is meant to exclude terms such as add, subtract, multiply, divide, number, numeral, triangle, fraction, percent, series, or other mathematical terms used in common parlance. The table below gives examples of some uncommon terms which might be found in fourth-grade tests.

1. factor	8. negative numbers	15. radius
2. addend	9. even or odd numbers	16. circumference
3. average	10. line segments	17. altitude
4. product	11. place values	18. area
5. sum	12. hundreds place	19. lengths
6. divisor	13. diameter	20. perimeters
7. empty set	14. intersection of two lines	21. volume

Any items which involve uncommon terms are classified at Level 12 of Factor III, no matter what additional operations might be involved. The rationale for giving priority to Level 12 is that unless the term is known, the problem is not solvable no matter what additional operations might be called for. It should be noted that this category is for mathematical terms and should <u>not</u> be confused with the next to last level of Factor II, which is for units of measurement and their definitions. Several examples now follow:

A. Three boys were 58, 61, and 66 inches tall. To find their average height, you can

a) use 61 as the average
b) add the heights together
c) add the heights and multiply by 3
d) add the heights and divide by 3

B. In which of the following does the numeral 6 have the greatest place value?

a) 2.639
b) 65.13
c) 56.47
d) 4.376

C. In which sentence is ☐ the sum?

a) 14 - 6 = ☐
b) 9 + 3 = ☐
c) ☐ + 5 = 15
d) 4 + ☐ = 10

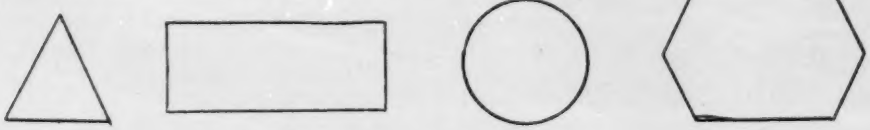
D. Which triangle is similar to triangle KLM?

a)
b)
c)
d)

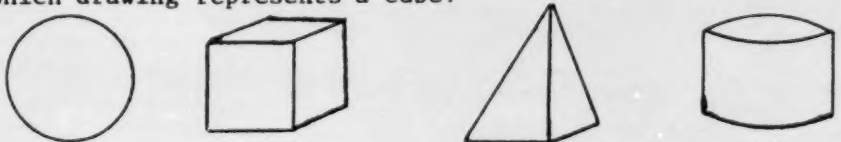
E. In which numeral is 5 both the hundreds place and the ones place?

a) 4,755
b) 4,575
c) 4,557
d) 5,475

F. Which of these figures has the least number of line segments?



G. Which drawing represents a cube?



In item A, the answer is the definition of an average. Even if the correct answer to this problem were the actual numerical value of the average, the item would still be classified at this level since to perform the mathematical operations to find the answer, the definition of an average known. To answer items C, D, E, and G, the definitions of sum, similar, hundreds place and ones place, and cube must be known. For item B, you need to know the definition of place value but, additionally, the operation of ordering is involved, i.e., recognizing that 60 > 06 > 0.6 > 0.0006. However,

the item is classified at Level 12 (rather than Level 11) since the meaning of place value is essential to understanding what is to be ordered. To find the answer to item F, the number of line segments for each figure must be counted and then those counts ordered, but before this can be done, one must know what is to be counted, i.e., what a line segment is.

Conventions for Using the Taxonomy for Factor III

1. If an item involves several operations which can be classified at more than one level of Factor III, the item should be classified at the highest level (where order is as specified in the taxonomy in Figure 1, i.e., the further down the list the higher the priority).
2. When classifying problems involving the solution of an algebraic sentence, classify according to the operation necessary to solve the equation, i.e., the operation opposite the one stated in the algebraic sentence.
3. If an item asks for a fraction of something or involves multiplying a fraction and a whole number (e.g., $\frac{1}{4} \times 8 = ?$), the item is classified as multiplication and not division.
4. If a whole number is added to or subtracted from a fraction, the whole number is assumed to be -- or, more precisely, can be readily converted into -- a fraction with the same denominator as the other fraction in the problem, and so the item would not be classified as add or subtract fractions without common denominators.
5. For problems involving graphs where all that is required for the answer is to look up a value on the graph, the operation is identify equivalents.
6. In graph problems, if the only operation involved is multiplying the

number of units times the value of the unit, the operation is identify equivalents.

Use of the Taxonomy

The foregoing sections described in detail and with examples the three factors defining the taxonomy and their corresponding levels. In order to use the taxonomy to classify items, each item should be read and then separately classified according to each of the three factors. The intersection of the selected levels for each of the three factors defines the cell into which the item falls. For coding purposes, the level (by number) of a given item on each of the three factors should be indicated on the code sheet (see Figure 2 for the code sheet and Figure 3 for the accompanying sheet indicating the numbers of the factor levels and their names). A single code sheet can be used for 40 items. Using the level numbers for an item, it is possible to plot the distribution of items across the individual cells of the matrix representing the taxonomy (as given in Figure 1). For items classified under identify terms or alternative number systems or essential units of measurement or geometric figures or others, the specific terms or units of measurement or geometric figures, etc. should also be recorded on the code sheet.

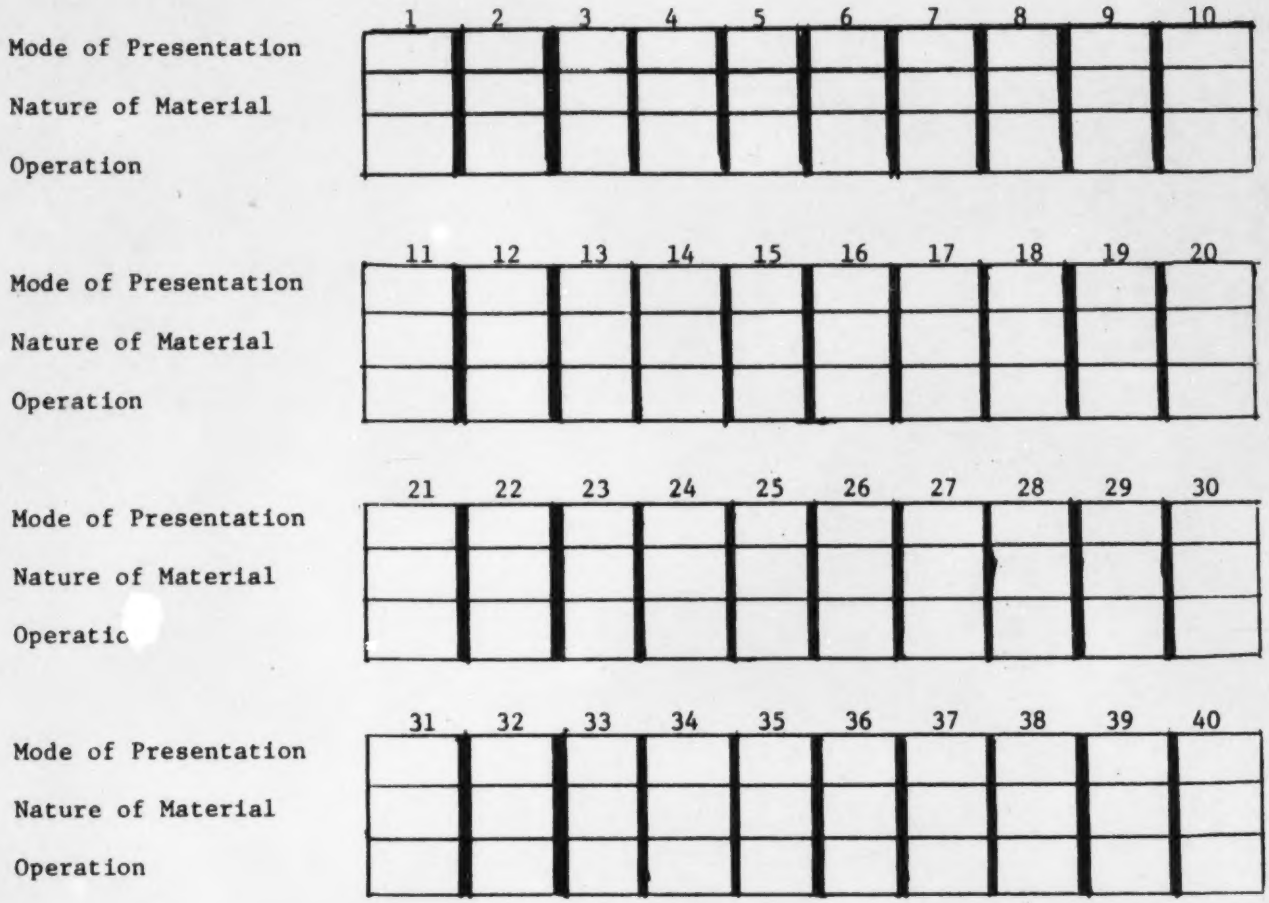
Listed below are several items and their proper classification. For some of the items -- those for which particular difficulty in classification is anticipated -- a detailed explanation of the classification is given. The items here were not chosen to be a random sample of items found on standardized tests, but rather a carefully selected set, representative of items which are difficult to classify.

Figure 2

Test _____ Classifier _____

Form _____ Date _____

Subtest _____


1 2 3 4 5 6 7 8 9 10
Mode of Presentation
Nature of Material
Operation
11 12 13 14 15 16 17 18 19 20
Mode of Presentation
Nature of Material
Operation
21 22 23 24 25 26 27 28 29 30
Mode of Presentation
Nature of Material
Operatic
31 32 33 34 35 36 37 38 39 40
Mode of Presentation
Nature of Material
Operation

Use this space to specify the Specific Content of *ed Items

Item #	Specific Content

Figure 3

CLASSIFICATION LEVELS

FACTOR I: Mode of Presentation

1. Graphs, Figures, Tables, or Physical Objects
2. Operation Specified
3. Operation Not Specified (story problems)

FACTOR II: Nature of Material

Whole Numbers
1. Single Digits
2. Single and Multiple Digits
3. Multiple Digits

Fractions
4. Single Fractions
5. Multiple Fractions

6. Decimals
7. Percents
8. Alternative Number Systems*
9. Place Value

Sentences
10. Number Sentences
11. Algebraic Sentences

12. Essential Units of Measurement*
13. Geometric Figures*
14. Other*

FACTOR III: Operations

1. Add
2. Subtract Without Borrowing
3. Subtract With Borrowing
4. Add or Subtract Fractions Without Common Denominator
5. Multiply
6. Divide Without Remainder
7. Divide With Remainder
8. Combination
9. Grouping
10. Identify Equivalents
11. Identify Rule (order)
12. Identify Terms*

*Be sure to Identify Specifics on Attached Page.

1. Which exercise below has as an answer a negative number?

a) 2 + 6
b) 6 - 2
c) 6 + 2
d) 2 - 6

Factor I = Operation specified
Factor II = Number sentence
Factor III = Identify term

2. Which number sentence below is false?

a) 3 tens + 8 ones = 8 tens and 3 ones
b) 30 + 9 = 20 + 19
c) 2 tens + 15 ones = 3 tens + 5 ones
d) 5 tens - 5 ones = 4 tens + 5 ones

Factor I = Operation specified
Factor II = Number sentences (although the item deals with place value as well, it is classified as number sentences since this level receives higher priority in the taxonomy; see Convention 1, p. 19)
Factor III = Identify equivalents (the problem concerns recognizing numerals expressed in different terms as being equivalent)

3. Which has the smallest value?

a) 1/2
b) 1/3
c) 60%
d) 75%

Factor I = Operation specified (see Convention 3, p. 5)
Factor II = Percentages (see Convention 1, p. 19 since both fractions and percentages are involved. Also note Convention 6, p. 7, which suggests that the alternatives must be used in classifying this item)
Factor III = Identify order

4. Which is the longest distance?

a) 1 ft. 3 in.
b) 16 in.
c) 14 in.
d) 1 ft. 6 in.

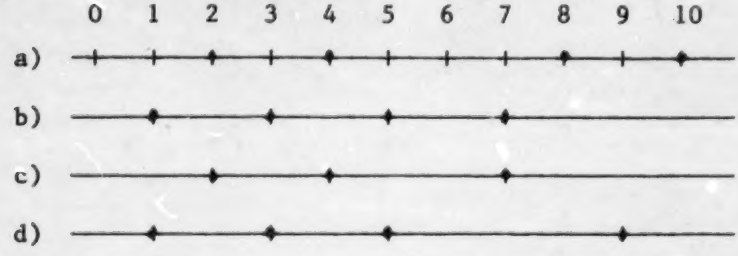
Factor I = Operation specified (see Convention 3, p. 5)
Factor II = Units of measurement (conversion of feet to inches is necessary)
Factor III = Identify order

5. 80,000 + 5,000 + 40 =

a) 85,040
b) 85,400
c) 80,540
d) 854

Factor I = Operation specified
Factor II = Place value (although whole numbers are involved, the emphasis is on place value)
Factor III = Identify equivalents (although addition is involved explicitly, the main emphasis is in recognizing the two forms for writing the numeral as equivalent)

6. Which of these is the graph of a set of even numbers?



Factor I = Graphs, figures, etc.
Factor II = Single and multiple digits
Factor III = Identify terms (even numbers)

7. Which line segment is nearest three inches long?

a) ——————————————————
b) ——————————————
c) ——————————
d) ——

Factor I = Graphs, figures, etc.
Factor II = Units of measurement (you must be able to transform the concept of 3 inches to its spatial equivalent in terms of length)
Factor III = Identify equivalents (although the term "line segment" is used, the context makes the meaning obvious)

8. Estimate which of these multiplications will give the greatest product.

a) 501 x 4
b) 410 x 4
c) 390 x 4
d) 499 x 4

Factor I = Operation specified
Factor II = Number sentence (this is so since you do not need to find the actual answers to solve the problem, only recognize which number sentence would yield the largest product)
Factor III = Identify terms (the answer depends on knowing the term product)

9. About how many ounces does a quart of milk weigh?

a) 2
b) 30
c) 200
d) 75

Factor I = Operation specified
Factor II = Unit of measurement
Factor III = Identify equivalents

10.

3	8	13	18	

What number comes next?

a) 23
b) 19
c) 20
d) 22

Factor I = Operation not specified (see Convention 4, p. 6)
Factor II = Multiple digits (see Convention 9, p. 20, and two of the digits are multiple)
Factor III = Identify rule

11. On pet day at school, nine children brought 3 dogs, 4 rabbits, 2 turtles, and 2 hamsters. How many pets did we have altogether?

a) 7
b) 20
c) 11
d) 9

Factor I = Operation not specified (story problem)
Factor II = Single digits (the intermediate equation is $3 + 4 + 2 + 2 = x$, which is a number sentence involving single digits)
Factor III = Add

12. $6 + \square = 9$

a) 4
b) 15
c) 3
d) NG

Factor I = Operation specified (see Convention 2, p. 5)
Factor II = Algebraic sentence
Factor III = Subtract without borrowing (see Convention 2, p. 32)

13. What part of this figure is shaded?

a) 1/2
b) 1/3
c) 1/4
d) 1/5

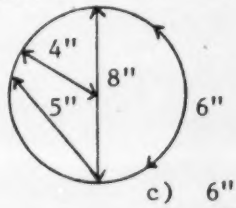
Factor I = Graphs, figures, etc.
Factor II = Single fractions
Factor III = Identify equivalents

14. There were 13 children waiting to cross the road. Eight of them ran across. How many were still waiting to cross the road?

a) $\square \times 8 = 13$
b) $13 - 8 = \square$
c) $13 + \square = 18$
d) $13 + 8 = \square$

Factor I = Operation not specified
Factor II = Number sentence (some of the distractors are algebraic sentences, but they are not necessary for the solution of this item and so are not used in classification)
Factor III = Identify equivalents

15. How long is the diameter of the circle shown below?



a) 4"
b) 5"
c) 6"
d) 8"

Factor I = Graphs, figures, etc.
Factor II = Geometric figures
Factor III = Identify terms

16. On Saturday both yards were full. There were 36 cars in all; 19 of them were in Clay's yard. How many were in John's yard?

a) 13
b) 17
c) 23
d) NG

Factor I = Operation not specified
Factor II = Algebraic sentence (the intermediate equation is $36 = 19 + x$, see Convention 10, p. 21)
Factor III = Subtract with borrowing (see Convention 2, p. 32)

Practice Items

The following is a series of practice items. Read each item and classify it according to each of the three factors. After you have completed all of the items turn to the section giving the answers and check your classifications.

1. $\frac{1}{2} + \frac{1}{4} =$

   a) 1/8
   b) 1/6
   c) 3/4
   d) 8

2. 35 2/3

   $\underline{-\quad 12}$

   a) 20 2/3
   b) 22 3/4
   c) 23 1/3
   d) 23 2/3

3. $3.00\overline{)24.00} =$

   a) .08
   b) .6
   c) 7
   d) 8

4. What is the largest number that can be represented using the numerals 9, 1, and 3 once each?

   a) 139
   b) 391
   c) 913
   d) 931

5. What should replace the $\square$ in the number sentence

   $5 + 6 = 6 + (\square + 3)$?

   a) 2
   b) 3
   c) 5
   d) 6

6. $4 + (6 + 11) = (4 + \square) + 11$

   Which numeral belongs in the box?

   a) 10
   b) 17
   c) 6
   d) 15

7. Mr. and Mrs. Smith and their three children are having a picnic lunch. They have invited an uncle, two aunts, and five cousins. If they all come, how many people will be at the picnic?

a) 13
b) 14
c) 11
d) 8

8. Rose has 18 problems to work. She has finished 6 of them. How many more does she have to work?

a) 8
b) 10
c) 24
d) 12

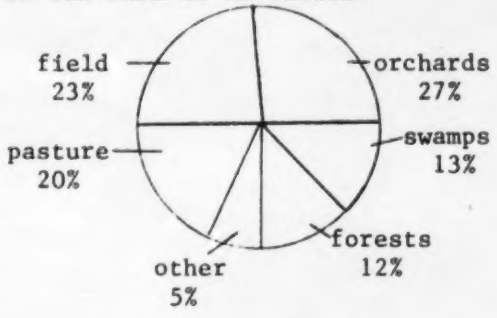
9. What change should Nancy get back? She gave a quarter and 3 dimes to pay for paper that cost 47¢. The tax is 2¢.

a) 6¢
b) 8¢
c) 11¢
d) 3¢

10. In Pete's room the temperature was 72^{o}, but the cellar temperature was only 47^{o}. How many degrees colder was the cellar?

a) 34
b) 25
c) 24
d) NG

11. What percentage of the farm is in fields?



a) 10%
b) 12%
c) 23%
d) 25%

12. The largest percentage of the farm is used for

a) orchards
b) forests
c) fields
d) pastures

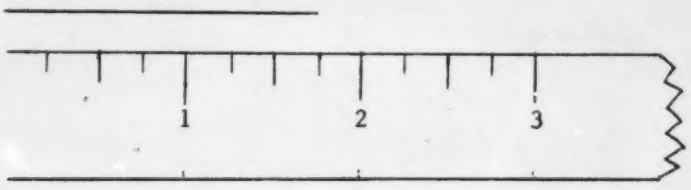
13. The numeral 9 in 19 stands for

a) 9 ones
b) 9 tens
c) 19 ones
d) 19 tens

14. In which exercise below is 8 an addend?

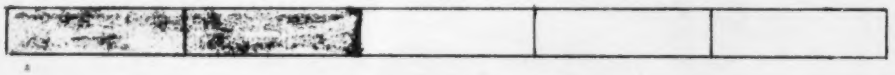
a) 8 - 2 = 6
b) 2 x 4 = 8
c) 8 + 2 = 10
d) 8 - 2 = 4

15. In the picture below, how long is the line segment above the ruler?



a) one and one-half inches
b) one and three-quarters inches
c) two inches
d) two and one-quarter inches

16. What fractional part of this figure is shaded?



a) 1/3
b) 2/5
c) 1/2
d) 2/6

17. In which of these numerals does the 3 have the least value?

a) 300
b) 630
c) 939
d) 103

18. Which set has both odd and even numbers?

a) (9, 11, 15, 3, 5)
b) (6, 12, 4, 2, 10)
c) (8, 6, 9, 3, 4)
d) (21, 13, 11, 9, 5)

19. VI =

a) 11
b) 4
b) 51
d) 6

Key for the Practice Items

Item	Factor I	II	III
1	2*	5	4
2	2	4	2
3	2	6	6
4	2	9	11
5	2	11	9
6	2	11	9
7	3	1	1
8	3	11	2
9	3	12	8
10	3	3	3
11	1	7	10
12	1	7	11
13	2	9	10
14	2	10	12
15	1	4	10
16	1	4	10
17	2	9	11
18	2	1	12
19	2	8	10

*Level numbers as given in the manual for each of the three factors.